The Infinite Clone Paradox

It is often claimed that in a truly infinite universe, anything which can happen *must* happen, and not just once but an infinite number of times. If this is true then in an infinite universe there must be (or must have been) an infinite number of copies of me writing this particular essay – and an infinite number of identical copies of you reading it. For many people this conclusion is distinctly unpalatable. But if you reject the conclusion you must reject the premise – i.e. you must conclude that the universe is spatially finite.

This is a staggeringly bold conclusion. We have plenty of evidence that our universe is temporally finite in one direction, but there is absolutely nothing in any of our current observations which would lead us to conclude that the universe is anything but infinite beyond the limits of our vision. The James Webb Space Telescope is showing us galaxies whose light has taken nearly the age of the universe to reach us – but that does not mean that galaxies whose light has yet to reach us do not exist. The fact is – the question is undecided. It is perfectly possible that the universe is spatially infinite. We just don't know.

So does that mean that it is perfectly possible that infinite clones do actually exist? Let's have a look at the actual argument.

Consider the decimal digits of π. π is believed to be what is called a 'normal' number[1]. What this means is that, in any sufficiently large sample, the frequency of the digits 0 – 9 is consistent with the digits being totally random. If this is true then it follows that any finite sequence of numbers can be found an infinite number of times somewhere in the digits of π. I am told that the sequence 123456789 first appears at the 523,551,502nd digit (but I am not going to check this!). The argument goes as follows: suppose that the former sequence occurs once and once only in the digits of π; then we know that the digit which follows the sequence 12345678 elsewhere will never be 9. This contradicts the assumption that the number passes all tests for randomness. It follows that there must be an infinite number of such sequences.

Let's apply the same argument to the universe. To do this we must make three assumptions:

1. The universe is spatially infinite
2. The laws of physics are the same everywhere
3. The universe is quantised – that is to say, in any given finite volume of space there is only a finite number of possible combinations of atoms and molecules.

The first is a given. There is plenty of evidence that the second is true within the observable part of the universe and I will go along with the idea that it is true even if the universe is infinite. The flaw in the argument must therefore be in the third assumption.

First I shall accept that the universe is spatially quantised on the scale of the Planck length. The Planck length is defined as $\sqrt{\frac{\hbar G}{c^3}}$ and is equal to 1.6×10^{-35} m. In every cubic metre there are therefore 4×10^{105} discrete points. A large but finite number. To keep things simple, let us suppose that our cubic metre contains nothing but photons and that each point either contains a photon or not. The number of possible states of this cubic metre will therefore be $2^{4\times10^{105}} \approx 10^{10^{105}}$ – a staggeringly large but still finite number.

So far so good.

Now let us consider the fact that each of these photons can have a different energy.

That's all right, you say, energy is quantised too just like length and time.

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1 Although the great majority of mathematicians believe this to be true, I don't think it has been proved.

Energy is quantised, true; but not like length and time.

The Planck unit of mass is defined as $\sqrt{\frac{\hbar c}{G}}$ and is equal to 2×10^{-8} kg. Unlike the Planck length and Planck time, this is a surprisingly large value being about the mass of a speck of dust. The Planck energy is even larger as it is the Planck mass multiplied by c^2. It works out to be about 2 thousand MJ. This is the same as the kinetic energy of a car travelling at 70 mph!

It is immediately obvious that this is not the *minimum* quantity of energy which an object can have as it is perfectly possible to drive a car at a slower speed than this. Nor is it a *maximum*. So what is the significance of the Planck energy?

Let's go back to our photons. The energy of a photon is related to its wavelength λ by the formula $E_{\text{photon}} = \frac{hc}{\lambda}$ and you will not be surprised to hear that when the energy of a photon is equal to the Planck energy, the wavelength of the photon is equal to the Planck length.

So what do we mean when we say 'energy is quantised'? In the context of atoms and spectra it means that photons emitted by an excited atom can only have certain discrete values. But in the context of a free photon, as far as we know, a photon can have any energy at all.

A plausible hypothesis might be that the energy of a free photon must be such that its wavelength is a whole number of Planck lengths[2]. If this is true then the *shortest* wavelength which a photon can have is, of course, the Planck length itself and the *maximum* energy which a photon can have is, therefore, the Planck energy.

Since, on this hypothesis, photons are allowed to have any wavelength which is a *multiple* of the Planck length, as the wavelength increases, the energy *decreases*. In other words, a photon can have any energy equal to the Planck energy *divided by* a whole number. What this means for us is that, even if energy is quantised in this way, there are always *an infinite number of possible states* for a photon (or indeed any particle) to be in.

In other words, assumption 3 is false and at a stroke, the argument for an infinite number of clones collapses. Even if the universe is quantised, there are always an infinite number of ways a finite region of space can be arranged. Just because the laws of physics have allowed a brain capable of writing this essay to appear on an insignificant planet 13 billion years after the Big Bang, it does not mean that a similar brain has or ever will appear somewhere else even if the universe is infinite.

For which we might be grateful, you may say.

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2 As far as I know, there is no evidence for this.