

Particularly Pretty Perfect Patio Pavings

Perfect Patio Pavings

In an earlier article I proposed an interesting set of tiles which make what I call Perfect Patio Pavings. The defining characteristics of such a paving are:

- It consists of at least one of each of a monomer, dimer, tetramer and hexamer.
- It is a Tatami Tiling i.e. it has no + crossings where 4 tiles share a common vertex
- It has no cracks i.e. you cannot draw a line across it which splits it into two
- It has no shared edges (with the exception of monomer/dimer)

Particularly Pretty Pavings

To qualify as being Pretty, hexamers and tetramers must not touch, and to be Particularly Pretty, hexamers and tetramers must not touch the edge of the paving either.

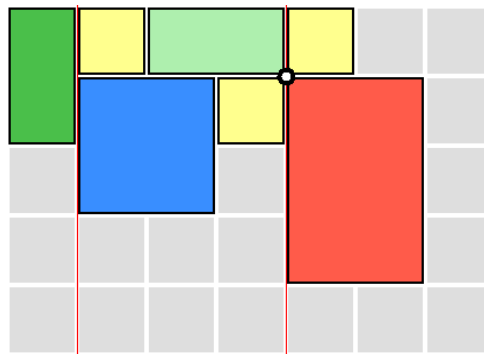
The question is – what is the smallest rectangular Particularly Pretty Perfect Patio Paving?

In the illustrations which follow, the tiles which are shown in bold colour are those which are placed initially. Tiles which follow as a consequence are shown in lighter colours.

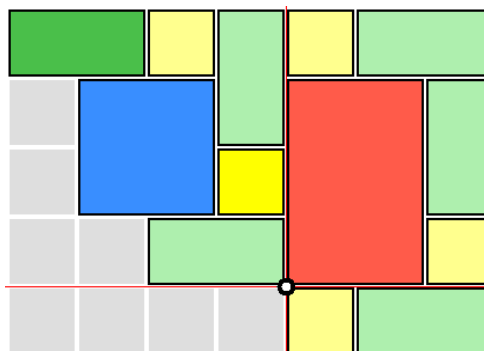
The 7×5 rectangle

There is only one way of positioning a tetramer and a hexamer in a 7×5 rectangle and wherever you place the dimer in the top left hand corner you run into trouble.

If you try a vertical dimer, this forces a crossing next to the hexamer:

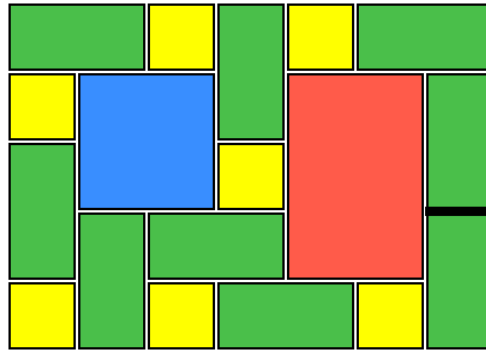


If you try a horizontal dimer, you can get right round the hexamer but then run into trouble at the bottom.



If we complete the area round the tetramer, we can move the problem to the right hand end of the

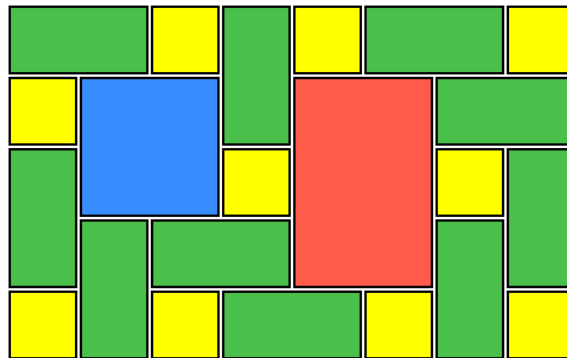
rectangle like this:



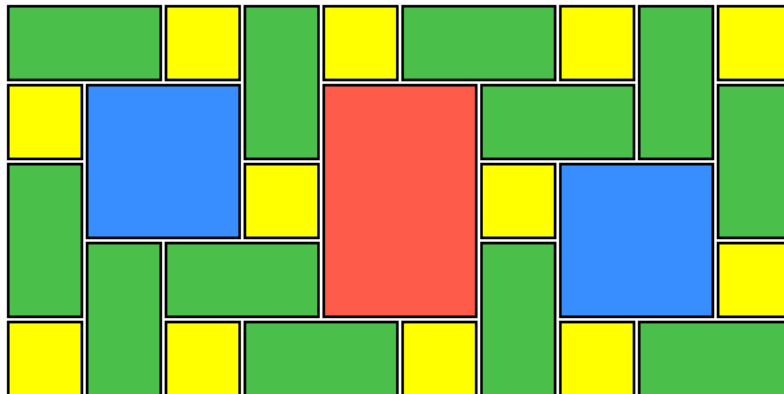
but there is a solution to this and that is to use an 8×5 rectangle.

The 8×5 rectangle

This is the solution we seek. The smallest Particularly Pretty Perfect Patio Paving!

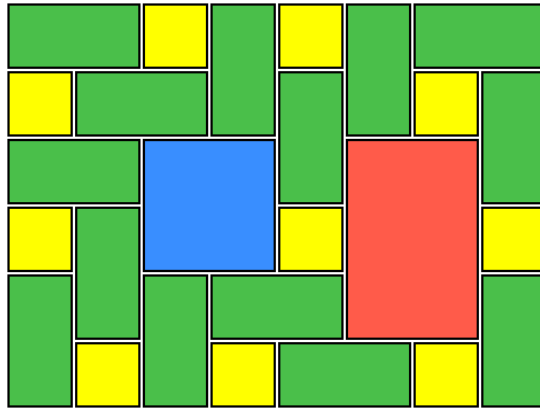


Noting that the tiles round the hexamer have rotational symmetry, it is obvious that we can construct a particularly pleasing 10×5 rectangle with 2 tetramers.



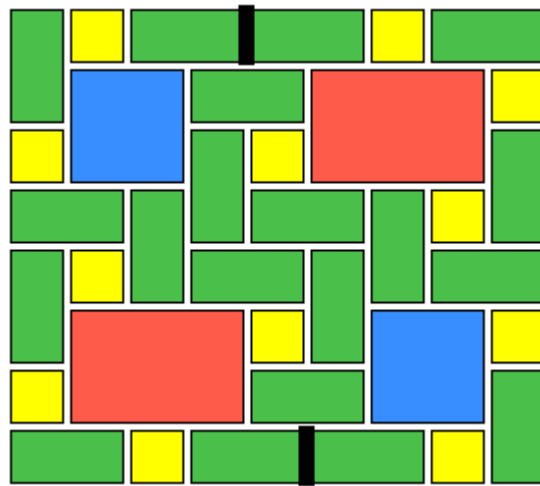
The 8×6 rectangle

Of the two large tiles it is the tetramer which is the bigger problem but there is a way of terminating a 6 unit wide rectangle with a tetramer and, happily, there is a way of finishing the rectangle off in 8 units using a hexamer:



Pavings with rotational symmetry

Here is a rather nice paving; unfortunately it does not qualify as a 'Pretty' paving because of the butting dimers but I like it all the same.



If you prefer a more random look how about this one for the most pleasing particularly pretty perfect patio paving ever?

